Flow of Dilute Polymer Solutions in Rough Pipes

MICHAEL POREH*
The University of Iowa, Iowa City, Iowa

A simplified model is developed to describe the effects of boundary roughness on drag reduction achieved by polymer additives. The model is based on the assumption that the effect of the roughness is similar for flows with and without polymers, and can be used for both uniform and nonuniform roughness. Predictions of friction coefficients by means of the model are in reasonable agreement with experimental results.

Introduction

THE use of polymers to reduce friction losses has been intensively investigated during the last decade, but most of the investigators have been concerned with flows past smooth boundaries. The question of drag reduction in the case of rough boundaries is of practical importance since most commercial pipes and surfaces of marine vehicles usually have some degree of roughness. Experiments with rough pipes[1-3] have shown that polymers are less effective in this case and that drag reduction tends to be zero as the roughness is increased. No theory has been offered to explain these observations.

A simplified analytical model describing the combined effect of uniform roughness and polymer solutions on the flow is proposed herein. The effect of nonuniform roughness, with and without polymers, is then related to the case of uniform roughness.

Flow of Dilute Polymer Solutions in Smooth Pipes

Although the exact mechanism of drag reduction is not fully understood, it is apparent from velocity-distribution measurements that drag reduction is associated with an increased thickness of the viscous sublayer and that the structure of the flow outside the wall region is hardly changed in dilute polymer solutions. It is also evident that drag reduction occurs only when the shear stress near the wall exceeds a critical value. Measurements in smooth pipes have led Meyer[4] to suggest that the mean longitudinal velocity u in dilute polymer solutions is described by the logarithmic equations

$$u/V^* = A \log(zV^*/\nu) + B + \Delta u^+ \tag{1}$$

and

$$\Delta u^+ = \alpha \log(V^*/V^*_{crit}) \tag{2}$$

where z is the distance from the wall, α is a concentration dependent parameter, $V^* = (\tau_\omega/\rho)^{1/2}$ is the shear velocity, V^*_{crit} is the shear velocity at the onset of drag reduction, and A, B are the constants used in the Newtonian case without polymer, $A = 5.75, B = 5.5$.

Equation (1) is assumed to be valid for $V^* > V^*_{crit}$ whereas for $V^* < V^*_{crit}$, $\Delta u^+ \equiv 0$. Equation (1) may also be written as

$$u/V^* = A \log[(zV^*/\nu)(V^*/V^*_{crit})^{\alpha/A}] + B \tag{3}$$

Integrating Eq. (1) over the pipe cross section, an approximate expression for the Darcy-Weisbach friction coefficient f is obtained:

$$1/f^{1/2} = \bar{A} \log Re(f)^{1/2} + \bar{B} + \Delta u^+/(8)^{1/2} \tag{4}$$

The integration gives $\bar{A} = 2.03$ and $\bar{B} = -0.9$; however, $\bar{A} = 2.0$ and $\bar{B} = -0.8$ give better agreement with the data and are usually assumed.

According to Eq. (2), the value of Δu^+ depends on two characteristic parameters of the polymer solution α and V^*_{crit}. Data collected by different investigators, with supposedly the same polymer solutions, however, do not always yield identical values of α or V^*_{crit}. One reason for the large scatter in the measured values of α and V^*_{crit} is undoubtedly the different characteristics of polymers which have the same brand name. In addition, polymer solutions can be degraded during the preparation of the solution or the experiment itself. The success of models used to predict the values of α and V^*_{crit} theoretically have so far been limited, and thus, it is required to determine them experimentally.[5-9]

Flow of Newtonian Fluids in Rough Pipes

Like polymers, surface roughness leaves the structure of the flow away from the wall unaltered, and the velocity-defect law remains valid in both cases.[10] It is therefore possible to express the velocity profiles of a Newtonian fluid near rough boundaries by the equation

$$u/V^* = A \log(zV^*/\nu) + B - F \tag{5}$$

where F is the downward shift of the logarithmic profile as a result of surface roughness. The dimensionless parameter F is found to be a function of the Reynolds number of the roughness kV^*/ν where k is the characteristic size of the roughness. Its exact form depends, however, on the shape of the roughness.

Figure 1, taken from Schlichting,[10] describes the measurements of Nikuradse for pipes with uniform sand roughness. The broken line in Fig. 1 describes the following approximation of F:

$$F(w) = 0, \; w < 3.35 \tag{6a}$$

$$F(w) = 0.26\,(w - 3.35) - 0.0026\,(w - 3.35)^2, \quad 3.35 < w < 20 \tag{6b}$$

$$F(w) = 5.75 \log[w - 2.0 - 17.4/(w)^{1/2}] - 3.0, \quad 20 < w \tag{6c}$$

where $w = kV^*/\nu$.

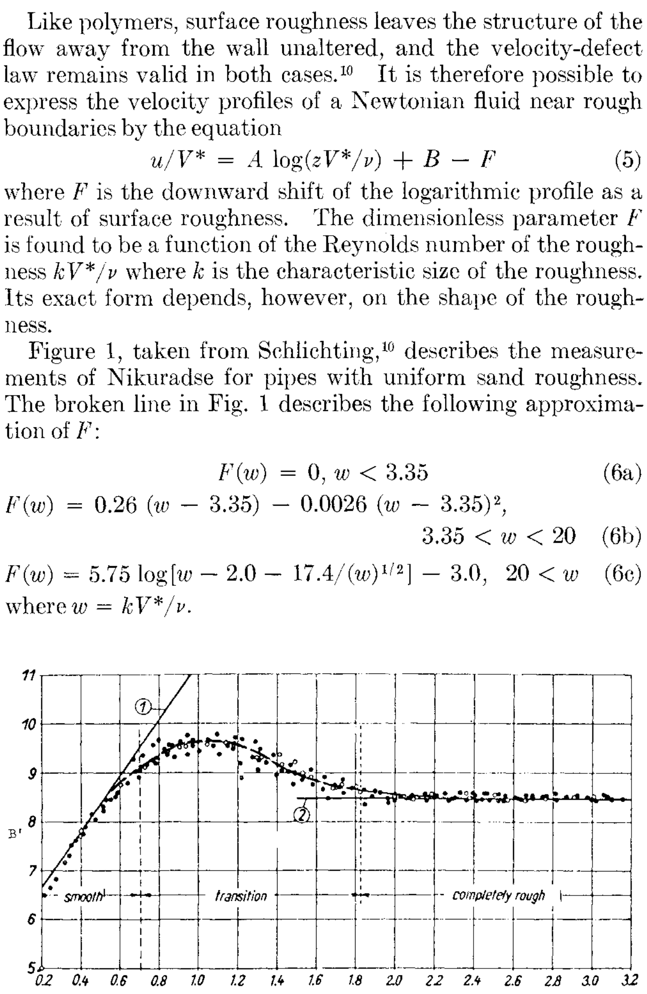

Fig. 1 Nikuradse's measurements in pipes with uniform sand roughness (according to Schlichting[10]); $B' = 5.75 \log (kV^*/\nu) + 5.5 - F$.

Received April 17, 1970; revision received August 14, 1970. The work was supported by Office of Naval Research, Contract Nonr-1611(03). The writer wishes to thank J. G. Spangler for sending him the original data published in Ref. 1, and L. Landweber for his interest in the work and helpful suggestions.

* Visiting Professor, Institute of Hydraulic Research (on leave from Technion—Israel Institute of Technology).

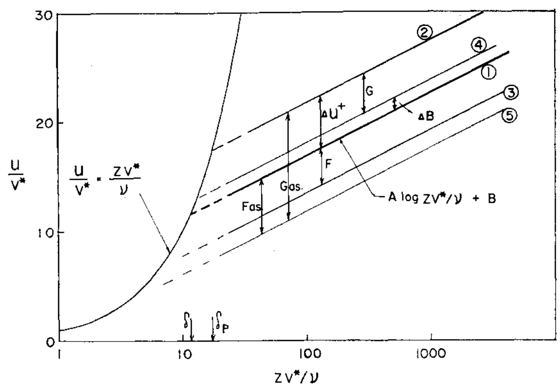

Fig. 2 Shift of the logarithmic profile due to roughness and polymer additives; curve 1 flow in smooth pipes without polymers, curve 2 flow in smooth pipes with polymers [Eqs. (1) and (2)], curve 3 flow in rough pipes without polymers [Eq. (5)], curve 4 flow in rough pipes with polymers [Eqs. (16) and (17)], curve 5 flow in very rough pipes with and without polymers [$G_{as} = \Delta u^+ - F_{as}$, Eq. (8)].

The asymptotic form of F for very large values of kV^*/ν is

$$F_{as} = A \log(kV^*/\nu) + C \quad (7)$$

where $C = -3.0$. Thus, the velocity profile for large kV^*/ν is given by

$$u/V^* = A \log(z/k) + B - C \quad (8)$$

and is independent of the viscosity.

In the region where $F_{as} > 0$, Eq. (5) may be written in the form

$$u/V^* = A \log(zV^*/\nu) + B - F_{as} \cdot R(kV^*/\nu) \quad (9)$$

where

$$R(kV^*/\nu) = F(kV^*/\nu)/F_{as} \quad (10)$$

The Reynolds number kV^*/ν may be expressed in terms of the ratio of k to a nominal sublayer thickness. The nominal sublayer thickness δ can be defined by the intersection of Eq. (1) and the equation $u/V^* = zV^*/\nu$. Accordingly, for Newtonian fluids without polymers

$$\delta V^*/\nu = A \log(\delta v^*/\nu) + B \quad (11)$$

which gives, for $A = 5.75$ and $B = 5.5$

$$\delta V^*/\nu = 11.6 \quad (12)$$

and

$$kV^*/\nu = (\delta V^*/\nu) \cdot (k/\delta) = 11.6\, k/\delta \quad (13)$$

Thus, Eq. (5) may be written as

$$u/V^* = A \log(zV^*/\nu) + B - F_{as} \cdot p(k/\delta) \quad (14)$$

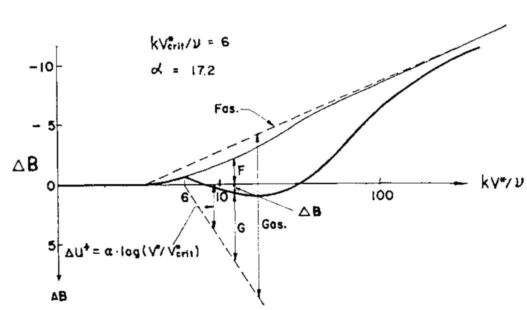

Fig. 3 Variation of ΔB with kV^*/ν ($kV^*_{crit}/\nu = 6$ and $\alpha = 17.2$).

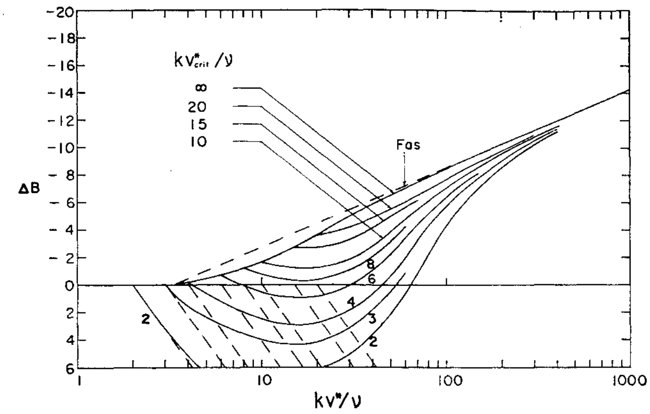

Fig. 4 Variation of ΔB for $\alpha = 17.2$.

where

$$p(k/\delta) = 0, \quad k/\delta < 0.29 \quad (15a)$$

$$p(k/\delta) = \{0.16[11.6(k/\delta) - 3.35] - 0.0026[11.6(k/\delta) - 3.35]^2\}/[5.75 \log(11.6\, k/\delta) - 3.0], \quad 0.29 < k/\delta < 1.72 \quad (15b)$$

$$p(k/\delta) = \{5.75 \log[11.6\, k/\delta - 2.0 - 17.4/(11.6\, k/\delta)^{1/2}] - 3.0\}/[5.75 \log(11.6\, k/\delta) - 3.0], \quad 1.72 < k/\delta \quad (15c)$$

Model for Flows of Polymer Solutions in Pipes with Uniform Roughness

Since neither rough boundaries nor polymer additives affect the structure of the turbulent flow away from the wall, it is reasonable to assume that the combined effect of both will not alter the velocity defect law either. It is therefore possible to express the velocity distribution (see Fig. 2) as

$$u/V^* = A \log(zV^*/\nu) + B + \Delta B \quad (16)$$

or

$$u/V^* = A \log[(zV^*/\nu)(V^*/V^*_{crit})^{\alpha/A}] + B - G \quad (17)$$

As already pointed out by Spangler[1] there is no drag reduction in very rough pipes, and the velocity profile is therefore described by Eq. (9). This indicates the asymptotic value of G in Eq. (17)

$$G_{as} = A \log[(kV^*/\nu)(V^*/V^*_{crit})^{\alpha/A}] - 3.0 \quad (18)$$

Thus, the function G in Eq. (17) varies from zero at small k/δ and approaches G_{as} as k/δ becomes large. Since Eqs. (5) and (7) can be considered the special case of Eqs. (17) and (18) when $\alpha = 0$; the assumption will be made that, similar to Eq. (14),

$$u/V^* = A \log[(zV^*/\nu)(V^*/V^*_{crit})^{\alpha/A}] - G_{as} \cdot p(k/\delta) \quad (19)$$

where G_{as} is given by Eq. (18), p is given by Eq. (15) and δ is

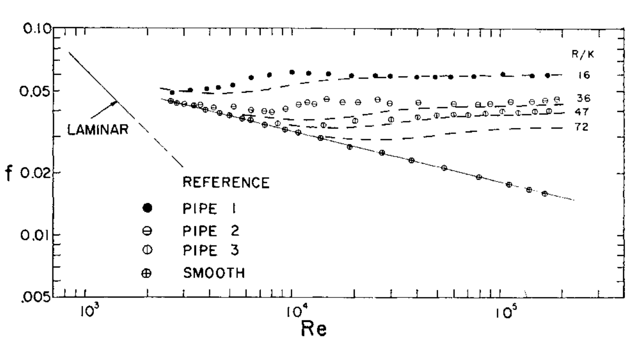

Fig. 5 Friction coefficients without polymers (experimental data of Spangler[1]).

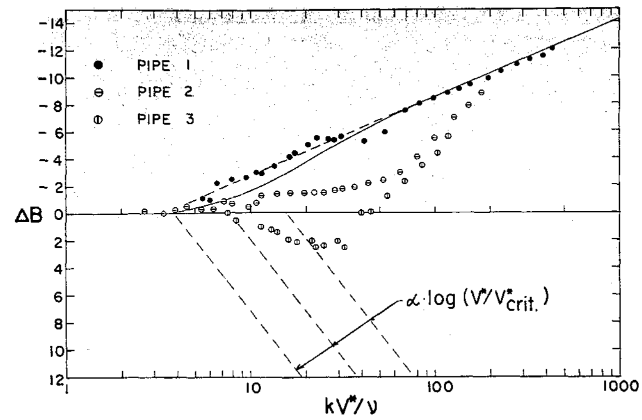

Fig. 6 Variation of ΔB according to Spangler.[1]

determined by

$$\delta V^*/\nu = A \log(\delta V^*/\nu) + B + \alpha\ (\log V^*/V^*_{crit}) \quad (20)$$

This satisfies the condition that Eqs. (19) and (20) reduce to Eqs. (14) and (12) when $\alpha = 0$. The parameter ΔB in Eq. (16) that describes the combined effect of the polymers and the roughness on the velocity profile (see Fig. 2) is thus given by

$$\Delta B = \alpha \log(V^*/V^*_{crit}) - G_{as}\cdot p(k/\delta) = \Delta u^+ - G_{as}\cdot p(k/\delta) \quad (21)$$

The value of $\delta V^*/\nu$ for polymer solutions is determined by α and V^*/V^*_{crit} and thus, ΔB is a function of the dimensionless parameters α, kV^*/ν, and kV^*_{crit}/ν. Figure 3 described the variation of ΔB according to Eq. (21) as a function of kV^*/ν for $kV^*_{crit}/\nu = 6$ and $= 17.2$. Since k, V^*_{crit}, and α are constants for a given pipe and a particular polymer solution, an increase of kV^*/ν in this figure corresponds to an increase in the velocity. When $kV^*/\nu < kV^*_{crit}/\nu$, the curve of ΔB for the polymer solution coincides with the corresponding curve for the flow of a Newtonian fluid; it is zero up to $kV^*/\nu = 3.35$ and negative thereafter. At $kV^*/\nu > kV^*/\nu_{crit}$, the value of ΔB increases because of the effect of the polymers. In this case, it is positive over a certain range of kV^*/ν indicating friction smaller than for a Newtonian fluid in a smooth pipe. At large kV^*/ν, the curve approaches the asymptotic curve for a completely rough flow. The broken line $\alpha\cdot\log(V^*/V^*_{crit})$ in this figure designates the value of ΔB for the flow of this polymer solution in a smooth pipe.

Similar curves for various values of kV^*_{crit}/ν and $\alpha = 17.2$ are given in Fig. 4. It is seen from this figure that for $kV^*_{crit}/\nu > 15$ drag reduction is very small and is limited to a narrow range of shear stresses. For kV^*/ν larger than 100, the drag reduction achieved is small even for efficient polymers with small V^*_{crit}.

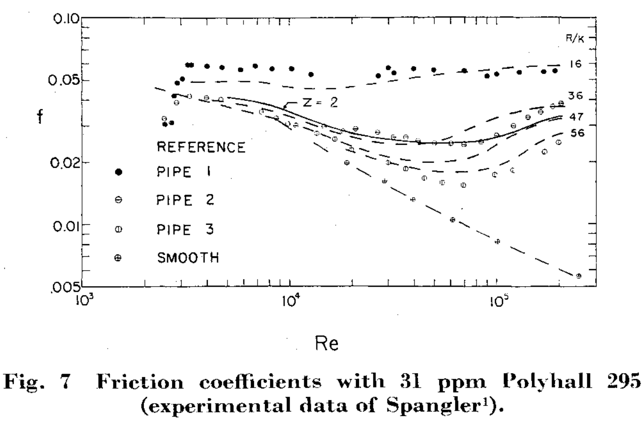

Fig. 7 Friction coefficients with 31 ppm Polyhall 295 (experimental data of Spangler[1]).

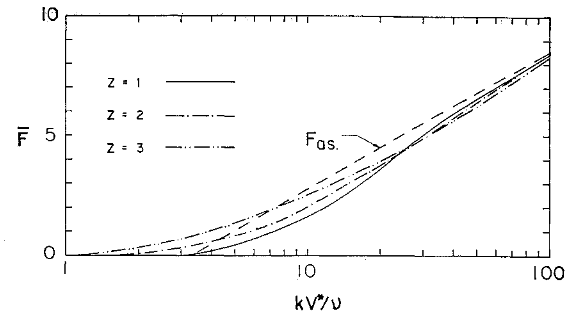

Fig. 8 The effect of roughness nonuniformity on $\bar{F}$.

Comparison with Experimental Results

Unfortunately, it is not possible to determine the values of α and V^*_{crit} for all the published data. Spangler[1] has reported a large number of measurements with a solution of Polyhall 295 in a smooth pipe and in three rough pipes. Polyhall 295, a product of Stein-Hall and Co., is an efficient drag reducing polymer with a molecular weight of 5–6 millions.

The roughness was produced by threading the inside of brass tubes with specially-made taps. On the basis of his measurements he concluded that for the particular concentration tested, $c = 31$ ppm, $\alpha = 17.2$, and $V^*_{crit} = 0.0968$ fps (2.95 cm/sec). The roughnesses in these pipes which we shall designate by 1, 2, and 3, were designed to give $R/k = 18$, 36, and 72 which correspond to $kV^*_{crit}/\nu \cong 16$, 8, and 4, respectively.

Figure 5 summarizes Spangler's measurements of the friction factor f without polymers and compares them with results from Nikuradse's measurements shown by broken lines. The measurements with pipe 2 are quite close to Nikuradse's line for $R/k = 36$ except for a deviation of approximately 20% at low Reynolds numbers. The data from pipe 1 deviate slightly from Nikuradse's line for $R/k = 18$. A better fit was obtained using $R/k = 16$ which corresponds to $kV^*_{crit}/\nu \cong 18$. The measurements in pipe 3 deviate considerably from the corresponding curve for $R/k = 72$ and instead follow the curve for $R/k = 47$. These measurements demonstrate clearly the fundamental difficulty encountered in predicting friction losses in rough pipes. The value of k for a particular type of roughness cannot be predicted with great accuracy. Even when an equivalent value of k is determined experimentally by measurements in the region $kV^*/\nu > 200$, deviations of more than 20% in f may be obtained at the lower-velocity range.

Spangler's measurements with Polyhall 295 are summarized in Figs. 6 and 7. Figure 6 was presented by Spangler who used the estimated values for R/k of 18, 36, and 47. The shape of the experimental curves in this figure is similar to

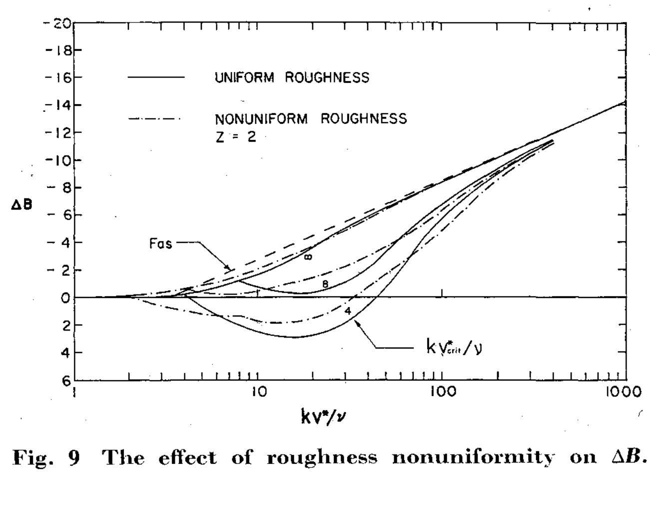

Fig. 9 The effect of roughness nonuniformity on ΔB.

that of the theoretical curves presented in Fig. 4. The data from pipe 1 show only small drag reduction near $kV^*/\nu = 50$ as suggested by the proposed model for this range of kV^*_{crit}/ν. The values of kV^*_{crit}/ν corresponding to $R/k = 18$ and 16 are approximately 16 and 18.

The data from pipe 2 ($kV^*_{crit}/\nu = 8$) follow more or less the shape of the corresponding theoretical line. The theoretical values of ΔB are found, however, to be slightly smaller than the experimental values near $kV^*/\nu = 20$ and larger than the experimental values near $kV^*/\nu = 80$.

The measurements in pipe 3 appear to be close to the theoretical curve for $kV^*_{crit}/\nu = 5$ which corresponds to $R/k = 56$.

The values of f calculated using the proposed model are compared with the measurements of f in Fig. 7. The difference between the theoretical values, which are described by broken lines, and the experimental ones are only slightly larger than the discrepancies found for flows without polymers (see Fig. 5).

Extension of the Model to Nonuniform Roughness

The derivation of the function p in the proposed model for uniform roughness is based on the assumption that $F(kV^*/\nu)$ in Eq. (5) is smaller than F_{as} and that the ratio F/F_{as} is finite for every value of kV^*/ν. This assumption is satisfied by Nikuradse's data as well as by the measurements of Hama[11] in boundary layers on uniformly roughened flat plates. However, it has been shown by Colebrook[12] that $F(kV^*/\nu)$ in commercial pipes with nonuniform roughness is not necessarily zero in the region $kV^*/\nu < 3.35$, although its asymptotic value is the same as for uniform sand roughness. A simplified model which explains the effect of roughness nonuniformity on F is proposed in this section. Using this model, it is possible to extend to nonuniform roughness the previously proposed method for estimating drag reduction.

Consider a surface with n sizes of roughness elements $k_1, \ldots, k_n$. The velocity profile in this case can be described by

$$u/V^* = A \log(zV^*/\nu) + B - \bar{F}(\bar{k}V^*/\nu) \tag{22}$$

where $\bar{k}$ is the equivalent size of the nonuniform roughness. It is proposed to estimate the function $\bar{F}$ as follows:

$$\bar{F}(\bar{k}V^*/\nu) = \sum_{i=1}^{n} a_i \cdot F(k_i V^*/\nu) \tag{23}$$

where F is given by Eq. (21), and a_i is the relative effective influence of the corresponding size. The coefficients a_i are positive and

$$\sum_{i=1}^{n} a_i = 1 \tag{24}$$

The equivalent roughness $\bar{k}$ is determined at large values of kV^*/ν where $\bar{F}$ is equal to F_{as}. Accordingly,

$$F_{as}(\bar{k}V^*/\nu) = \sum_{i=1}^{n} a_i F_{as}(k_i V^*/\nu) \tag{25}$$

or

$$\log(\bar{k}V^*/\nu) - 3.0 = \sum_{i=1}^{n} a_i[(\log(k_i V^*/\nu) - 3.0)] \tag{26}$$

It follows that

$$k = k_1^{a_1} \cdot k_2^{a_2} \cdot \ldots \cdot k_n^{a_n} \tag{27}$$

Consider for example a blend of two roughness elements k_1 and k_2 with $a_1 = a_2 = 0.5$ and $k_2 = Z \cdot k_1$, where Z is a parameter describing the roughness nonuniformity. It follows from Eq. (27) that

$$\bar{k} = Zk_1 = k_2/Z \tag{28}$$

When k_i and a_i are known, the function $\bar{F}(\bar{k}V^*/\nu)$ can be calculated from Eq. (23). Figure 8 describes the shape of $\bar{F}$ for $Z = 2$ and 3. When $Z > 1$ the effect of roughness is recognized for $\bar{k}V^*/\nu < 3.35$. Physically, this demonstrates the effect of the larger elements. On the other hand, the effect of the nonuniformity at large values of $\bar{k}V^*/\nu$ is rather small. The curve for $Z = 3$ is quite similar to those obtained in commercial pipes.[12]

Let us examine now the effect of drag-reducing polymers in rough pipes of nonuniform roughness. Using this model with $Z = 2$, the values of ΔB have been calculated and plotted in Fig. 9 for $\bar{k}V^*_{crit}/\nu = 4$, 8 and ∞. Again, the effect of the roughness becomes apparent at values of $\bar{k}V^*/\nu < 3.35$. It is seen that near $\bar{k}V^*/\nu = 20$, ΔB is smaller when $Z = 2$ than it is when $Z = 1$; while near $\bar{k}V^*/\nu = 80$, the mixed roughness gives more drag reduction than uniform roughness with the same $\bar{k}$. As pointed out earlier, the deviation of the experimental data from the theoretical curves for uniform roughness ($Z = 1$) are in the same direction. The deviation of Spangler's measurements with water at low Reynolds numbers also suggests that the threads produced an effect which is better described with the mixed roughness model. The values of f for pipe 2 with $R/k = 36$ and $Z = 2$ have been calculated and are shown in Fig. 7 by a solid line. This line is a better approximation of the experimental data than the broken line calculated with $Z = 1$.

The function $\bar{F}$ can also be calculated for a continuous distribution of roughness. In this case,

$$\bar{F}(\bar{k}V^*/\nu) = \int_0^\infty a(k)F(kV^*/\nu)dk \tag{30}$$

where $a(k)$ is an effective weight distribution which satisfies the condition

$$\int_0^\infty a(k)dk = 1 \tag{31}$$

The equivalent roughness $\bar{k}$ is determined in this case by the equation

$$\log\bar{k} = \int_0^\infty a(k) \log k dk \tag{32}$$

Conclusions

The lack of detailed theories for drag reduction and for turbulent flows in rough pipes makes it impossible to derive a rigorous theory to describe the effects of polymers on flow in rough pipes. The simplified model proposed in this work is based on the assumption that the effect of the relative roughness size is similar for flows with and without polymers. The model appears to be successful in describing, at least qualitatively, the experimental results. The deviation of the experimental results from the theoretical calculations with the model is of the same order of magnitude as the one obtained in flows without polymers.

References

[1] Spangler, J. G., "Studies of Viscous Drag Reduction with Polymers Including Turbulence Measurements and Roughness Effects," *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, 1969, p. 131.

[2] Brandt, H., McDonald, A. T., and Boyle, F. W., "Turbulent Skin Friction of Dilute Polymer Solutions in Rough Pipes," *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, 1969, p. 159.

[3] White, H., "Some Observations on the Flow of Characteristics of Certain Dilute Macromolecular Solutions," *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, 1969, p. 297.

[4] Meyer, W. A., "A Correlation of the Frictional Characteristics for Turbulent Flow of Dilute Non-Newtonian Fluids in Pipes," *American Institute Chemical Engineers Journal*, Vol. 12, No. 3, 1966.

[5] Elata, C., Lehrer, J., and Kahanovitz, A., "Turbulent Shear Flow of Polymer Solutions," *Israel Journal of Technology*, Vol. 4, No. 87, 1966.

[6] Poreh, M., Rubin, H., and Elata, C., "Studies in Rheology and Hydrodynamics of Dilute Polymer Solutions," Publication No. 126, 1969, Civil Engineering Dept., Technion, Haifa, Israel.

[7] Virk, P. S., "The Toms Phenomenon—Turbulent Pipe Flow of Dilute Polymer Solutions," Sc.D. thesis, Massachusetts Institute of Technology, 1966; also Virk, P. S. and Merril, G. W., *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, 1969, p. 107.

[8] Paterson, R. W., "Turbulent Flow Drag Reduction and Degradation with Dilute Polymer Solutions," TR, Contract N00014-67-A-0298-0002, 1969, Harvard University, Cambridge, Mass.

[9] Whitsitt, N. F., Harrington, L. J., and Crawford, H. R., "Effect of Wall Shear Stress on the Drag Reduction of Viscoelastic Fluids," *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, 1969, p. 265.

[10] Schlichting, H., *Boundary Layer Theory*, McGraw-Hill, 1969.

[11] Hama, F. R., "Boundary Layer Characteristics for Smooth and Rough Surfaces," *Transactions of the Society of Naval Architects and Marine Engineers*, Vol. 62, 1954.

[12] Colebrook, C. F., "Turbulent Flow in Pipes with Particular Reference to the Transition Region between Smooth and Rough Pipe Laws," *Journal of the Institution of Civil Engineers*, 1939; also *Engineering Hydraulics*, edited by H. Rouse, Chapter VI.

OCTOBER 1970 J. HYDRONAUTICS VOL. 4, NO. 4

Diffusion of a Rectilinear Vortex in a Weakly Viscoelastic Liquid

RICHARD C. LESSMANN*
University of Rhode Island, Kingston, R. I.

The classical problem of the decay of a rectilinear vortex because of the action of viscosity is extended to include the diffusion of vorticity in a weakly viscoelastic fluid. A second-order Rivlin-Ericksen stress tensor is used to model the viscoelastic nature of the flowfield; and the governing equations are solved by use of the Hankel transform. It is found that a "classical" vortex structure can exist in this type of fluid only if a certain constant in the stress tensor is positive, and, by recourse to kinetic theory, this constant is found to be negative for aqueous solutions of high molecular weight polymers.

Nomenclature

A = Avogadro's number
$\overset{(1)}{B}{}^{ij}, \overset{(2)}{B}{}^{ij}$ = contravariant Rivlin-Ericksen acceleration tensors
C = polymer concentration by weight
D/Dt = $\partial/\partial t + \mathbf{u}\cdot\nabla$ = material derivative
$\hat{e}_r,\hat{e}_\theta,\hat{e}_z$ = unit vectors in cylindrical polar coordinates
$\mathbf{F}$ = surface force vector acting on a fluid particle
g_{ij} = metric tensor
k = Hankel transform variable
K = Boltzmann's constant
M = molecular weight of polymer
N = number density of polymer
P = pressure
T = absolute temperature
t = time
$\mathbf{u}$ = vector fluid velocity
u^i = ith component of the contravariant velocity vector
x_1,x_2,x_3 = cylindrical polar coordinates corresponding to r, θ, z
β,γ = material constants in the viscoelastic stress tensor
Γ = circulation
δ_{ij} = Kronecker delta ($\delta_{ij} = 0$, $i \neq j$, $\delta_{ii} = 1$)
μ = absolute viscosity
$[\mu]$ = intrinsic viscosity
ν = kinematic viscosity
ρ = density
τ^{ij} = contravariant stress tensor
τ_1 = relaxation time
$\boldsymbol{\omega}$ = vorticity vector

Subscripts

$(\ )_v$ = viscous variable
$(\ )_{ve}$ = viscoelastic variable

Received December 1, 1969; revision received August 10, 1970. This work was performed while the author was employed by the Naval Underwater Weapons Research and Engineering Station, Newport, R. I.

* Assistant Professor, Department of Mechanical Engineering and Applied Mechanics. Associate Member AIAA.

Introduction

THE decay of a rectilinear vortex because of the action of viscosity is a problem of fundamental importance in classical hydrodynamics. The solution of this problem is extended here to include vortex diffusion in a weakly viscoelastic medium which is described by a second-order Rivlin-Ericksen stress tensor.

Interest in this problem is an outgrowth of the fact that recently much engineering effort has been expended in the study of dilute aqueous solutions of high molecular weight polymers. This class of fluids is important because of their ability to greatly reduce the drag of a submerged body in turbulent flow,[1] and because the presence of polymers in a laminar boundary layer tends to inhibit cavitation.[2]

Injection of such drag reducing additives into a boundary layer means that control surfaces and propulsers must function in weakly viscoelastic environments. All classical analyses of such things as propeller blades and stabilizing fins require a knowledge of the behavior of idealized vortices; hence it is of particular interest to understand the nature of vortex flow in this type of fluid.

The second-order Rivlin-Ericksen approximation to the stress tensor is used in this study for several reasons. Most important of these is that it probably represents the limit of tractability for an analytic investigation and because of this, it enjoys a wide application in the literature. In addition, it has been successful in demonstrating normal stress effects in